Quadratic Equations

- **Identification of quadratic equations**

Example: Check whether the following are quadratic equations or not.
(i) $(2x + 3)^2 = 12x + 3$
(ii) $x (x + 3) = (x + 1) (x – 5)$
Solution:
$$(\mathbf{i})\ (2x+3)^2 = 12x+3$$
$$\Rightarrow 4x^2 + 12x + y = 12x + 3$$
$$\Rightarrow 4x^2 + 6 = 0$$
It is of the form $ax^2 + bx + c = 0$, where $a = 4$, $b = 0$ and $c = 6$
Therefore, the given equation is a quadratic equation
$$(\mathbf{ii})\ x(x+3) = (x+1)(x-5)$$
$$\Rightarrow x^2 + 3x = x^2 + x - 5x - 5$$
$$\Rightarrow 7x + 5 = 0$$
It is not of the form $ax^2 + bx + c = 0$, since the maximum power (or degree) of equation is 1.
Therefore, the given equation is not a quadratic equation.

- **Express given situation mathematically**

Example 1:
An express train takes 2 hour less than a passenger train to travel a distance of 240 km. If the average speed of the express train is 20 km/h more than that of a passenger train, then form a quadratic equation to find the average speed of the express train?
Solution:
Let the average speed of the express train be x km/h.
Since it is given that the speed of the express train is 20 km/h more than that of a passenger train,
Therefore, the speed of the passenger train will be $x - 20$ km/h.
Also we know that $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$
Time taken by the express train to cover 240 km = $\frac{240}{x}$
Time taken by the passenger train to cover 240 km = $\frac{240}{x-20}$
And the express train takes 2 hour less than the passenger train. Therefore,

$$\frac{240}{x-20} - \frac{240}{x} = 2$$

$$\Rightarrow 240\left[\frac{x-(x-20)}{x(x-20)}\right] = 2$$

$$\Rightarrow 120\left(\frac{20}{x^2-20x}\right) = 1$$

$$\Rightarrow 2400 = x^2 - 20x$$

$$\Rightarrow x^2 - 20x - 2400 = 0$$

This is the required quadratic equation.

- **Solution of Quadratic Equation by Factorization Method**

  If we can factorize $ax^2 + bx + c = 0$, where $a \neq 0$, into a product of two linear factors, then the roots of this quadratic equation can be calculated by equating each factor to zero.

  Example:

  Find the roots of the equation, $2x^2 - 7\sqrt{3}x + 15 = 0$, by factorisation.

  Solution:

  $$2x^2 - 7\sqrt{3}x + 15 = 0$$
  $$\Rightarrow 2x^2 - 2\sqrt{3}x - 5\sqrt{3}x + 15 = 0$$
  $$\Rightarrow 2x\left(x - \sqrt{3}\right) - 5\sqrt{3}\left(x - \sqrt{3}\right) = 0$$
  $$\Rightarrow \left(x - \sqrt{3}\right)\left(2x - 5\sqrt{3}\right) = 0$$
  $$\Rightarrow \left(x - \sqrt{3}\right) = 0 \text{ or } \left(2x - 5\sqrt{3}\right) = 0$$
  $$\Rightarrow x = \sqrt{3} \text{ or } x = \frac{5\sqrt{3}}{2}$$

  Therefore, $\sqrt{3}$ and $\frac{5\sqrt{3}}{2}$ are the roots of the given quadratic equation.

- **Solution of Quadratic Equation by completing the square**

  A quadratic equation can also be solved by the method of completing the square.

  Example:

  Find the roots of the quadratic equation, $5x^2 + 7x - 6 = 0$, by the method of completing the square.

  Solution:

$$5x^2+7x-6=0$$
$$\Rightarrow 5\left[x^2+\frac{7}{5}x-\frac{6}{5}\right]=0$$
$$\Rightarrow x^2+2\times x\times\frac{7}{10}+\left(\frac{7}{10}\right)^2-\left(\frac{7}{10}\right)^2-\frac{6}{5}=0$$
$$\Rightarrow \left(x+\frac{7}{10}\right)^2-\frac{49}{100}-\frac{6}{5}=0$$
$$\Rightarrow \left(x+\frac{7}{10}\right)^2=\frac{169}{100}$$
$$\Rightarrow \left(x+\frac{7}{10}\right)=\pm\sqrt{\frac{169}{100}}=\pm\frac{13}{10}$$
$$\Rightarrow x+\frac{7}{10}=\frac{13}{10} \text{ or } x+\frac{7}{10}=-\frac{13}{10}$$
$$\Rightarrow x=\frac{13}{10}-\frac{7}{10}=\frac{3}{5} \text{ or } x=-\frac{13}{10}-\frac{7}{10}=-2$$

Therefore, –2 and $\frac{3}{5}$ are the roots of the given quadratic equation.

- **Quadratic Formula to find solution of quadratic equation:**

The roots of the quadratic equation, $ax^2+bx+c=0$, are given by, $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where $b^2-4ac\geq 0$

Example:

Find the roots of the equation, $2x^2-3x-44=0$, if they exist, using the quadratic formula.

Solution:

$$2x^2-3x-44=0$$

Here, a = 2, b = –3, c = –4

$$\therefore b^2-4ac=(-3)^2-4\times 2\times(-44)=9+352=361>0$$

The roots of the given equation are given by $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

$$\Rightarrow x=\frac{-(-3)\pm\sqrt{361}}{2\times 2}=\frac{3\pm 19}{4}$$
$$\Rightarrow x=\frac{3+19}{4}=\frac{11}{2} \text{ or } x=\frac{3-19}{4}=-4$$

The roots are –4 and $\frac{11}{2}$.

- **Nature of roots of Quadratic Equation**

For the quadratic equation, $ax^2+bx+c=0$, where $a \neq 0$, the discriminant 'D' is defined as $D=b^2-4ac$

The quadratic equation, $ax^2+bx+c=0$, where $a \neq 0$, has

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1. two distinct real roots, if $D=b^2-4ac>0$
2. two equal real roots, if $D=b^2-4ac=0$
3. has no real roots, if $D=b^2-4ac<0$

Example: Determine the nature of the roots of the following equations

(a) $2x^2+5x-117=0$

(b) $3x^2+5x+6=0$

Solution:

(a) Here, a = 2, b = 5, c = –117

$\therefore D=b^2-4ac=5^2-4\times 2\times(-117)=25+936=961>0$

Therefore, the roots of the given equation are real and distinct.

(b) Here, a = 3, b = 5, c = 6

$\therefore D=b^2-4ac=5^2-4\times 3\times 6=25-72=-47<0$

Therefore, the roots of the given equation are not real.